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ON THE RIEMANN PROBLEM FOR LIQUID OR GAS–LIQUID MEDIA

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SUMMARY

Self-similar solutions to the Riemann problem for water with the modified Tait equation of state are presented. The methods of Smoller for gas dynamics are employed to reduce the problem to the solution of a single non-linear equation. The same methods are used for solving the Riemann problem at a gas–water interface. In both cases the method of interval bisections affords a solution technique free of problems with convergence.

KEY WORDS Riemann problem Gas–liquid interface Modified Tait equation of state

1. INTRODUCTION

The solution of the Riemann problem in gas dynamics is well understood. A typical reference which spells out the complexity and solution techniques for this problem is the work of Sod.[1] However, for liquid media such a fortunate state generally does not exist. Where the most general[2] equation of state is employed, methods for solving the Riemann problem exactly are not yet known. The case where an approximate solution can be tolerated is investigated in Reference 3.

The work of Holl[4] establishes to a good approximation that a functional form equivalence exists between the wave relations for gas and water, where the modified Tait equation of state is employed.[5] This has led some investigators[6] to conjecture that under the transformation $P \mapsto P + B$, $\gamma \mapsto N$ (where B and N are parameters of the equation of state for water), solutions of the Riemann problem for gas dynamics map into solutions of the Riemann problem for water. Thus codes for computer solution of the Riemann problem for gas dynamics can be simply adapted for use in water.

The objective of this paper is to develop an exact Riemann solver for water. In particular, the Riemann problem at a gas–water interface will be solved both theoretically and numerically. By this means it becomes possible to compare results with that obtained by adapting to water a gas dynamics Riemann solver which is based upon the above mapping. Excellent agreement between results from the two approaches is obtained.

2. THE GAS–WATER RIEMANN PROBLEM

Consider an imaginary plane diaphragm separating a water state 1 from a gas state 4. If the diaphragm is suddenly ruptured, a rarefaction wave or a shock wave moves into the gas, separating it into states 3 and 4, while a rarefaction wave or a shock wave also moves into the

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water, separating it into states 1 and 2 (see Figure 1). State 2, which is water, is separated from state 3, which is gas, by a contact surface C across which pressure and velocity are continuous.

Now the Riemann problem which governs the nature of the solution after rupture of the diaphragm can be shown to have a self-similar solution (which is constant on straight lines through the initial discontinuity). The theory of self-similar solutions to the Euler equations guarantees that states 1–4 will have constant properties, the state boundaries will be linear and constant states are separated either by shock waves across which the Rankine–Hugoniot relations apply or by simple (rarefaction) waves across which isentropic flow relations hold.

The gas–water Riemann problem (R)

Given states 1 and 4, find the possible states 2 and 3 such that the total solution 1–4 satisfies the Euler equations of compressible flow, with gas constitutive relations on one side and water constitutive relations on the other side of the contact surface C.

Notes

1. The velocities in states 1 and 4 are not necessarily zero.
2. When the velocities in states 1 and 4 are zero, (R) represents the hydrodynamic shock tube problem with gas driver.

The solution to the Riemann problem (R) has been studied by Flores and Holt,[7] who reduce it to the simultaneous solution of several non-linear equations. For some cases difficulty in obtaining convergence is experienced when these equations are solved by iterative means.

In the present section it is shown that this Riemann problem can be reformulated in such a way that its solution can be reduced to the solving of one non-linear equation. This solution can be accomplished by the method of interval bisection and no difficulties with convergence are encountered.

Some terminology is necessary. According to Figure 1, we shall refer to the left-propagating wave front separating states 3 and 4 as a 1-wave, while the contact surface separating states 2

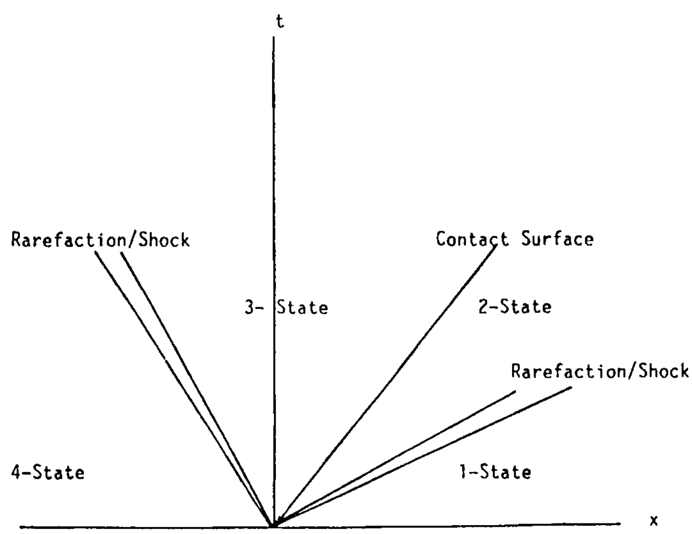

Figure 1. The Riemann problem

and 3 is a 2-wave and the right-propagating wave front separating states 1 and 2 is a 3-wave. Given a particular left state U_L, the set of corresponding right states U_R which are connected to this left state by either a shock, a contact surface or an expansion front will be referred to as 1-, 2- or 3-family. By searching for pressure, density and velocity relations across 1-, 2- or 3-rarefactions and 1-, 2- or 3-shocks, the Riemann problem (R) can be expressed in a form readily amenable to numerical solution.

For each particular family, generic left and right states U_L and U_R are assumed. If $P_3 < P_4$, the wave front between states 3 and 4 is a 1-rarefaction; if $P_3 > P_4$, the wave front between states 3 and 4 is a 1-shock. Similarly, if $P_2 > P_1$, the wave front connecting states 1 and 2 is a 3-shock; otherwise it is a 3-rarefaction. Since there must be a gas–water separation, the 2-family will always be a contact surface.

The Rankine–Hugoniot jump conditions

The Rankine–Hugoniot jump conditions across a shock wave in an ideal gas may be written as

$$\rho_1 u_1 v_1 - \rho_0 u_0 v_0 = P_0 - P_1, \tag{1}$$

$$\rho_1 v_1 = \rho_0 v_0 = m, \tag{2}$$

$$\frac{2}{\gamma - 1} C_1^2 + v_1^2 = \frac{2}{\gamma - 1} C_0^2 + v_0^2. \tag{3}$$

Here $v = u - W$, where W represents the speed of the shock, and subscripts 0 and 1 denote conditions on the left and right sides of the shock respectively.

For water equation (1) is modified to

$$\rho_1 u_1 v_1 - \rho_0 u_0 v_0 = \bar{P}_0 - \bar{P}_1, \tag{4}$$

whereas (3) is inapplicable. However, the modified Tait equation applies across either a shock or a rarefaction wave in the form

$$\bar{P}_1/\bar{P}_0 = (\rho_1/\rho_0)^N, \tag{5}$$

where N is a constant and $\bar{P} = P + B$.

Across an expansion wave the following Riemann invariants, valid for both gas and water, may be used:

$$u_1 \pm \frac{2}{\gamma - 1} C_1 = u_0 \pm \frac{2}{\gamma - 1} C_0, \tag{6}$$

where the plus sign refers to 1-waves and the minus sign to 3-waves. For water N replaces γ, with the speed of sound given by

$$C^2 = N\bar{P}/\rho.$$

3. CONTACT SURFACE RELATIONS

In this section we derive the one-parameter families of relation $U_R/U_L = f(x)$ which determine the set of right states U_R which are connected to a given left state U_L through a 1-, 2- or 3-wave. These relations can be used to solve the gas–water Riemann problem completely. Likewise, expressions for contact surface velocity and thermodynamic state emerge. For water the modified Tait equation of state

$$P = B[(\rho/\rho_0)^N - 1] \tag{7}$$

of course leads to the relation

$$\bar{P}/\bar{P}_0 = (\rho/\rho_0)^N, \tag{8}$$

which holds across a rarefaction or a shock wave.

It is assumed that the gas is ideal, so the eequation of state is given by

$$P = \rho RT. \tag{9}$$

This leads to

$$P/P_0 = (\rho/\rho_0)^\gamma \tag{10}$$

across a rarefaction wave, whereas the Rankine–Hugoniot relations (1)–(3) hold across shock waves.

As previously mentioned, there are three families of functions which are intermediaries in describing the relationship between the gas state 4 and the water state 1. The first and third families describe the gas and water media, with shock or rarefaction waves depending upon the relative pressure distribution of the two sides.

The relations across rarefaction waves for the first and third families will now be derived. For completeness the results for the gas dynamics case, although well known,[8] are also included.

Rarefaction waves for gas

The derivation of relations across waves in a gas is presented by J. Smoller.[8] He introduces a parameter x_1 by the relation

$$x_1 = -\ln(P_3/P_4) \geqslant 0. \tag{11}$$

For the 1-rarefaction wave in the gas Smoller derives the relations ($x_1 \geqslant 0$)

$$P_3/P_4 = e^{-x_1}, \tag{12}$$

$$\rho_3/\rho_4 = e^{-x_1/\gamma}, \tag{13}$$

$$\frac{u_3 - u_4}{C_4} = \frac{2}{\gamma - 1}\left(1 - \frac{C_3}{C_4}\right) = \frac{2}{\gamma - 1}(1 - e^{-\tau x_1}), \tag{14}$$

where

$$\tau = \frac{\gamma - 1}{2\gamma}. \tag{15}$$

Rarefaction waves for water

For water the 3-family rarefaction wave satisfies ($x_3 \leqslant 0$)

$$P_1/P_2 = e^{x_3}, \tag{16}$$

$$\frac{\bar{P}_1}{\bar{P}_2} = \frac{1 + B_1}{e^{-x_3} + B_1}, \tag{17}$$

where

$$\bar{P} = P + B, \tag{18}$$

$$B_1 = B/P_1. \tag{19}$$

Also,

$$\frac{\rho_1}{\rho_2} = \left(\frac{\bar{P}_1}{\bar{P}_2}\right)^{1/N} = \left(\frac{1 + B_1}{e^{-x_3} + B_1}\right)^{1/N}, \tag{20}$$

$$\frac{u_1 - u_2}{C_2} = \frac{2}{N - 1}\left[\left(\frac{1 + B_1}{e^{-x_3} + B_1}\right)^{\bar{\tau}} - 1\right], \tag{21}$$

with

$$\bar{\tau} = \frac{N - 1}{2N}. \tag{22}$$

Shocks in gas

Referring again to Reference 8, with $x_1 \leqslant 0$ and

$$\beta = \frac{\gamma + 1}{\gamma - 1} \tag{23}$$

the 1-family of right states connecting a given left state is given by

$$P_3/P_4 = e^{x_1}, \tag{24}$$

$$\frac{\rho_3}{\rho_4} = \frac{\beta + e^{x_1}}{1 + \beta e^{x_1}}, \tag{25}$$

$$\frac{u_3 - u_4}{C_4} = \frac{2\sqrt{\tau}}{\gamma - 1}\frac{1 - e^{-x_1}}{\sqrt{(1 + \beta e^{-x_1})}}. \tag{26}$$

Shocks in water

Similarly, with $x_3 \leqslant 0$ the 3-family shock wave relations for water can be written as

$$P_1/P_2 = e^{x_3}, \tag{27}$$

$$\frac{\bar{P}_1}{\bar{P}_2} = \frac{1 + B_1}{e^{-x_3} + B_1}, \tag{28}$$

$$\frac{\rho_1}{\rho_2} = \left(\frac{1 + B_1}{e^{-x_3} + B_1}\right)^{1/N}, \tag{29}$$

$$\frac{u_1 - u_2}{C_2} = \frac{1}{\sqrt{N}}\left(\frac{(\rho_1/\rho_2)(1 - P_2/P_1)}{(P_2/P_1 + B_1)(\rho_1/\rho_2 - 1)}\right)^{1/2} - \frac{1}{\sqrt{N}}\left(\frac{1 - P_2/P_1}{(P_2/P_1 + B_1)(\rho_1/\rho_2 - 1)(\rho_1/\rho_2)}\right)^{1/2}. \tag{30}$$

For the 2-family one should have the contact surface relations

$$P_2 = P_3, \qquad \rho_2/\rho_3 = e^{x}, \qquad u_2 = u_3. \tag{31}$$

Thus there emerge the combined one-parameter families for the gas–water Riemann problem which are listed below.

Combined one-parameter relations

1-family

$$P_3/P_4 = e^{-x_1}, \tag{32}$$

$$\frac{\rho_3}{\rho_4} = f_1(x_1) = \begin{cases} e^{-x_1/\gamma}, & x_1 \geq 0, \\ \dfrac{\beta + e^{x_1}}{1 + \beta e^{x_1}}, & x_1 \leq 0, \end{cases} \tag{33}$$

$$\frac{u_3 - u_4}{C_4} = h_1(x_1) = \begin{cases} \dfrac{2}{\gamma - 1}(1 - e^{-\tau x_1}), & x_1 \geq 0, \\ \dfrac{2\sqrt{\tau}}{\gamma - 1} \dfrac{1 - e^{-x_1}}{\sqrt{(1 + \beta e^{-x_1})}}, & x_1 \leq 0; \end{cases} \tag{34}$$

2-family

$$P_2/P_3 = 1, \tag{35}$$

$$\rho_2/\rho_3 = e^{x_2}, \tag{36}$$

$$u_2 = u_3; \tag{37}$$

3-family

$$P_1/P_2 = e^{x_3}, \tag{38}$$

$$\frac{\bar{P}_1}{\bar{P}_2} = \frac{1 + B_1}{e^{-x_3} + B_1}, \tag{39}$$

$$\frac{\rho_1}{\rho_2} = f_3(x_3) = \left(\frac{1 + B_1}{e^{-x_3} + B_1}\right)^{1/N}, \tag{40}$$

$$\frac{u_1 - u_2}{C_2} = h_3(x_3),$$

where

$$h_3(x_3) = \begin{cases} \dfrac{2}{N - 1}\left[\left(\dfrac{1 + B_1}{e^{-x_3} + B_1}\right)^{\bar{\tau}} - 1\right], & x_3 \geq 0, \\ \dfrac{1}{\sqrt{N}}\left(\dfrac{(\rho_1/\rho_2)(1 - P_2/P_1)}{(P_2/P_1 + B_1)(\rho_1/\rho_2 - 1)}\right)^{1/2} & \\ \quad - \dfrac{1}{\sqrt{N}}\left(\dfrac{1 - P_2/P_1}{(P_2/P_1 + B_1)(\rho_1/\rho_2 - 1)\rho_1/\rho_2}\right)^{1/2}, & x_3 \leq 0. \end{cases} \tag{41}$$

From the above relations one has

$$u_1 - u_4 = C_2 h_3(x_3) + C_4 h_1(x_1), \tag{42}$$

$$\frac{C_2}{C_1} = \left(\frac{1 + B_1}{e^{-x_3} + B_1}\right)^{\bar{\tau}}, \tag{43}$$

$$\frac{P_1}{P_4} = \frac{P_1}{P_2}\frac{P_2}{P_4} = e^{x_3 - x_1}, \tag{44}$$

$$\frac{\rho_1}{\rho_4} = \frac{\rho_1}{\rho_2}\frac{\rho_2}{\rho_3}\frac{\rho_3}{\rho_4} = f_3(x_3)e^{x_2}f_1(x_1), \tag{45}$$

$$x_2 = \ln\left(\frac{\rho_1}{\rho_4}[f_3(x_3)f_1(x_1)]^{-1}\right), \tag{46}$$

$$x_1 = x_3 + \ln(P_4/P_1). \tag{47}$$

Thus

$$\frac{u_1 - u_4}{C_1} = \frac{C_4}{C_1}h_1(x_1) + h_3(x_3)\left(\frac{1 + B_1}{e^{-x_3} + B_1}\right)^{(N-1)/2N} \tag{48}$$

By substituting (47) into (48), the ensuing equation can be solved iteratively by the method of interval bisection to obtain x_3. No convergence problems are experienced. The values of x_1 and x_2 follow from (46) and (47). Thus the gas–water Riemann problem has been solved. Equations (41) provide the contact surface velocity. Equation (38) yields the contact surface pressure.

4. SOLUTION OF THE RIEMANN PROBLEM FOR WATER

In order to apply Godunov's method to liquid media, the approach of Section 2 will now be used to develop the one-parameter families of shock and rarefaction relations which permit solution of the Riemann problem in water with the modified Tait equation of state:

1-family

$$P_3/P_4 = e^{-x_1}, \tag{49}$$

$$\frac{\bar{P}_3}{\bar{P}_4} = \frac{e^{-x_1} + B_4}{1 + B_4}, \tag{50}$$

$$\frac{\rho_3}{\rho_4} = f_1(x_1) = \left(\frac{e^{-x_1} + B_4}{1 + B_4}\right)^{1/N}, \tag{51}$$

where

$$B_4 = B/P_4, \tag{52}$$

$$\frac{u_3 - u_4}{C_4} = h_1(x_1),$$

$$h_1(x_1) = \begin{cases} \dfrac{2}{N-1}\left[1 - \left(\dfrac{e^{-x_1} + B_4}{1 + B_4}\right)^{\bar{\tau}}\right], & x_1 \geqslant 0, \\ \dfrac{1}{\sqrt{N}}\left(\dfrac{1 - P_3/P_4}{(1 + B_4)(\rho_3/\rho_4)(1 - \rho_3/\rho_4)}\right)^{1/2} \\ -\dfrac{1}{\sqrt{N}}\left(\dfrac{(1 - P_3/P_4)\rho_3/\rho_4}{(1 + B_4)(1 - \rho_3/\rho_4)}\right)^{1/2}, & x_1 \leqslant 0, \end{cases} \tag{53}$$

with

$$\bar{\tau} = \frac{N-1}{2N}; \tag{54}$$

2-family

$$P_2/P_3 = 1, \tag{55}$$

$$\rho_2/\rho_3 = e^{x_2}, \tag{56}$$

$$u_2 = u_3; \tag{57}$$

3-family

$$P_1/P_2 = e^{x_3}, \tag{58}$$

$$\frac{\bar{P}_1}{\bar{P}_2} = \frac{1 + B_1}{e^{-x_3} + B_1}, \tag{59}$$

$$\frac{\rho_1}{\rho_2} = f_2(x_3) = \left(\frac{1 + B_1}{e^{-x_3} + B_1}\right)^{1/N}, \tag{60}$$

$$\frac{u_1 - u_2}{C_2} = h_3(x_3),$$

where

$$h_3(x_3) = \begin{cases} \dfrac{2}{N-1}\left[\left(\dfrac{1 + B_1}{e^{-x_3} + B_1}\right)^{\bar{\tau}} - 1\right], & x_3 \geqslant 0, \\ \dfrac{1}{\sqrt{N}}\left(\dfrac{(\rho_1/\rho_2)(1 - P_2/P_1)}{(P_2/P_1 + B_1)(\rho_1/\rho_2 - 1)}\right)^{1/2} \\ -\dfrac{1}{\sqrt{N}}\left(\dfrac{1 - P_2/P_1}{(P_2/P_1 + B_1)(\rho_1/\rho_2 - 1)\rho_1/\rho_2}\right)^{1/2}, & x_3 \leqslant 0. \end{cases} \tag{61}$$

From the above relations one has

$$u_1 - u_4 = C_2 h_3(x_3) + C_4 h_1(x_1), \tag{62}$$

$$\frac{C_2}{C_1} = \left(\frac{1 + B_1}{e^{-x_3} + B_1}\right)^{\bar{\tau}}, \tag{63}$$

$$\frac{P_1}{P_4} = \frac{P_1}{P_2}\frac{P_2}{P_4} = e^{x_3 - x_1}, \tag{64}$$

$$\frac{\rho_1}{\rho_4} = \frac{\rho_1}{\rho_2}\frac{\rho_2}{\rho_3}\frac{\rho_3}{\rho_4} = f_3(x_3)e^{x_2}f_1(x_1), \tag{65}$$

$$x_2 = \ln\left(\frac{\rho_1}{\rho_4}[f_3(x_3)f_1(x_1)]^{-1}\right), \tag{66}$$

$$x_1 = x_3 + \ln(P_4/P_1). \tag{67}$$

Then

$$\frac{u_1 - u_4}{C_1} = \frac{C_4}{C_1}h_1(x_1) + h_3(x_3)\left(\frac{1 + B_1}{e^{-x_3} + B_1}\right)^{(N-1)/2N}. \tag{68}$$

By substituting (67) into (68), the ensuing equation can be solved iteratively by the method of interval bisection to obtain x_3. No convergence problems are experienced. The values of x_1 and x_2 follow from (66) and (67). Thus the Riemann problem for water has been solved. Equations (61) provide the contact surface velocity.

Special note

For water–water situations, use of the same value for B on the two sides of a contact surface will not permit a density discontinuity in crossing the contact surface! Thus, if it is assumed that B is constant, choosing $x_2 = 0$ and discarding (65) prohibits the appearance of a contact discontinuity.

5. A WATER-ADAPTED GAS DYNAMICS RIEMANN SOLVER

The relations given by (11)–(15) and (23)–(26) form the basis for solving the classical Riemann problem for the ideal gas (see Reference 1 for further amplification). Also, relations (16)–(22) and (27)–(31) allow the construction of a Riemann solver code for water, which has been accomplished during the course of this research.

However, in liquid media the omission of an expression for internal energy forces the Rankine–Hugoniot jump conditions to be employed in enthalpy form:

continuity

$$\rho_1 v_1 = \rho_2 v_2; \tag{69}$$

momentum

$$P_1 + \rho_1 v_1^2 = P_2 + \rho_2 v_2^2; \tag{70}$$

energy

$$h_1 + v_1^2/2 = h_2 + v_1^2/2. \tag{71}$$

Here, as before,

$$v = u - W. \tag{72}$$

Moreover, Holl[4] has shown that when the modified Tait equation of state is used, the enthalpy rise across a water shock, referred to in (71), can be well approximated by the relation

$$h_2 - h_1 = \frac{N}{N-1}\left(\frac{\bar{P}_2}{\rho_2} - \frac{\bar{P}_1}{\rho_1}\right). \tag{73}$$

Therefore, to a good approximation and subject to the assumptions of Holl's derivation,[4] equation (71) can be replaced with

$$\frac{1}{N-1} C_1^2 + \frac{v_1^2}{2} = \frac{1}{N-1} C_2^2 + \frac{v_2^2}{2}. \tag{74}$$

Now the jump conditions (69) and (70) hold regardless of whether the medium is gas or liquid. Furthermore, for the case of an ideal gas equation (71) is equivalent to

$$\frac{1}{\gamma-1} C_1^2 + \frac{v_1^2}{2} = \frac{1}{\gamma-1} C_2^2 + \frac{v_2^2}{2}. \tag{75}$$

It is noted that the respective formulae for the speed of sound for water and gas are

$$C^2 = N\bar{P}/\rho, \tag{76}$$

$$C^2 = \gamma P/\rho. \tag{77}$$

The conclusion is that the correspondence

$$\bar{P} \Leftrightarrow P, \; N \Leftrightarrow \gamma \tag{78}$$

transforms gas shock relations to water shock relations. Of course, the same observation holds regarding relations across rarefaction waves. The outcome is that the following theorem has been established.

Theorem 1

To a good approximation the correspondence $\bar{P} \Leftrightarrow P$, $N \Leftrightarrow \gamma$ transforms the solution of the Riemann problem for water with the modified Tait equation of state into the solution of the Riemann problem for the idea gas and conversely.

Lemma 1

Any computer code which solves the Riemann problem for the ideal gas can be adapted via equation (78) to approximation of the solution to the Riemann problem for the ideal water (modified Tait equation of state).

Note 1

For the Riemann problem at a gas–water interface it is shown in the sequel by numerical experimentation that the result of applying Theorem 1 and Lemma 1 leads to negligible difference in results from that of coding the *bonda fide* water Riemann solver determined by the relations of Section 3.

6. NUMERICAL RESULTS

In this section the method of Godunov[6] and the techniques of Sections 2–4 will be used to simulate the solution to a certain spherical gas–water Riemann problem (see also References 7 and 9). It is assumed that the Riemann problem results from the release into stagnant liquid of a sphere of gas initially at high temperature and pressure. Table I gives the values of the basic parameters of the problem.

In what follows, all graphical results are non-dimensionalized as follows. Let $(\tilde{\ })$ denote dimensional quantities and choose $h = 1$ ft as a characteristic length. All lengths are divided by the characteristic length $R = \tilde{R}/h$, velocities are divided by the speed of sound in undisturbed water, $v = v/a_w$; densities are given by $\rho = \tilde{\rho}/\rho_w$; pressures are given by $P = \tilde{P}/P'_0$, where $P'_0 = P_w + B$; $P_w = 1$ atm in undisturbed water; and time is divided by h/a_w, i.e. $t = \tilde{t}/(h/a_w)$. The space step is either $\Delta R = 0{\cdot}01$ (coarse grid) or $\Delta R = 0{\cdot}00125$ (fine grid). For stability the time step is chosen to satisfy the condition

$$\max(|v| + a)\Delta t/\Delta R < 1,$$

where a is the local sound speed.

The Godunov method[6] for solving the conservation law equations

$$U_t + F(U)_R = 0 \tag{79}$$

can be written as

$$U_j^{n+1} = U_j^n - \frac{\Delta t}{\Delta R}(F^*_{j+1/2} - F^*_{j-1/2}). \tag{80}$$

The flux $F^*_{j+1/2}$ is obtained through solving the Riemann problem(s)

$$U_t + F(U)_R = 0, \tag{81}$$

$$U(x, 0) = \begin{cases} U_L, & x < x_y \\ U_R, & x > x_y \end{cases} \tag{82}$$

over a uniform grid of mesh width ΔR. The Riemann problem (81), (82) is solved separately (i) in the gas region, (ii) in the water region and (iii) for variable-size cells adjacent to the contact surface. Source terms not indicated by (81) are accounted for by the splitting technique elaborated in Reference 1. Solution of the Riemann problem(s) (81), (82) is accomplished by the methods of Sections 2–4.

Results from numerical solution of the spherical gas–water Riemann problem are depicted in Figures 2 and 3. Two approaches are employed: (a) a *bona fide* Riemann solver is used for

Table I. Values of basic parameters

Initial charge radius	$\frac{1}{3}$ ft
Depth of charge centre	1 ft
Initial pressure of explosion gas	9000 atm
Initial temperature of explosion gas	2500 K
Specific heat ratio of explosion gas	1·4
Initial water pressure	1 atm
Modified Tait equation of state	$B = 3268$ atm, $\rho_0 = 1007$ kg m^{-3}, $N = 7$

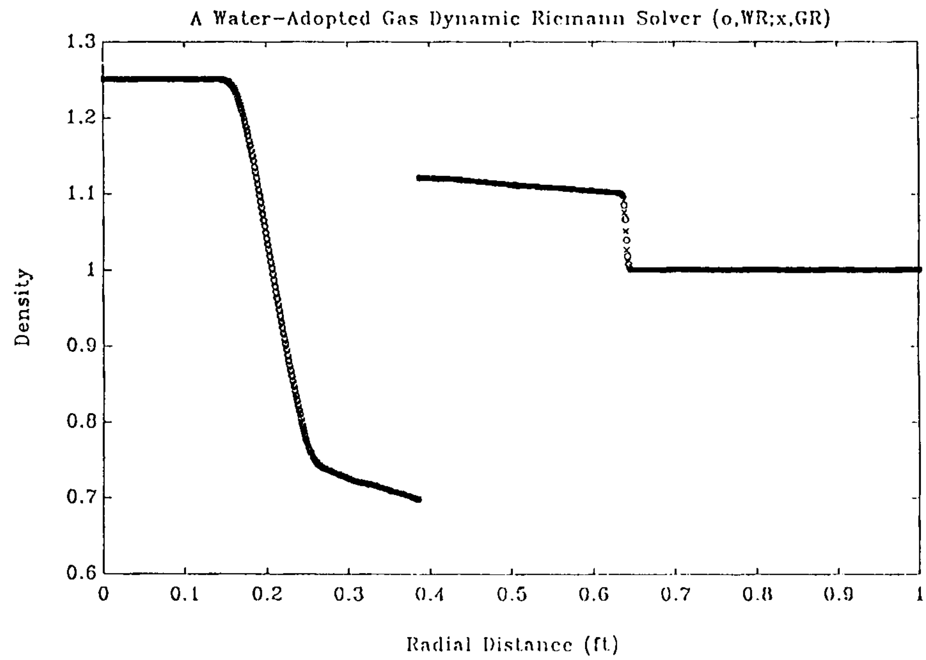

Figure 2. Comparison of results for an exact versus an approximate Riemann solver for water (density)

the water region; (b) the Riemann solver for the water side has been adapted from a gas dynamics Riemann solver according to the transformation of equation (78). The general quality of the results for density and pressure profiles shown in Figures 2 and 3 appears excellent. Mach number and velocity results (not shown) are equally pleasing. Moreover, it is clear that negligible error occurs as a result of the adaptation of the gas dynamics Riemann solver.

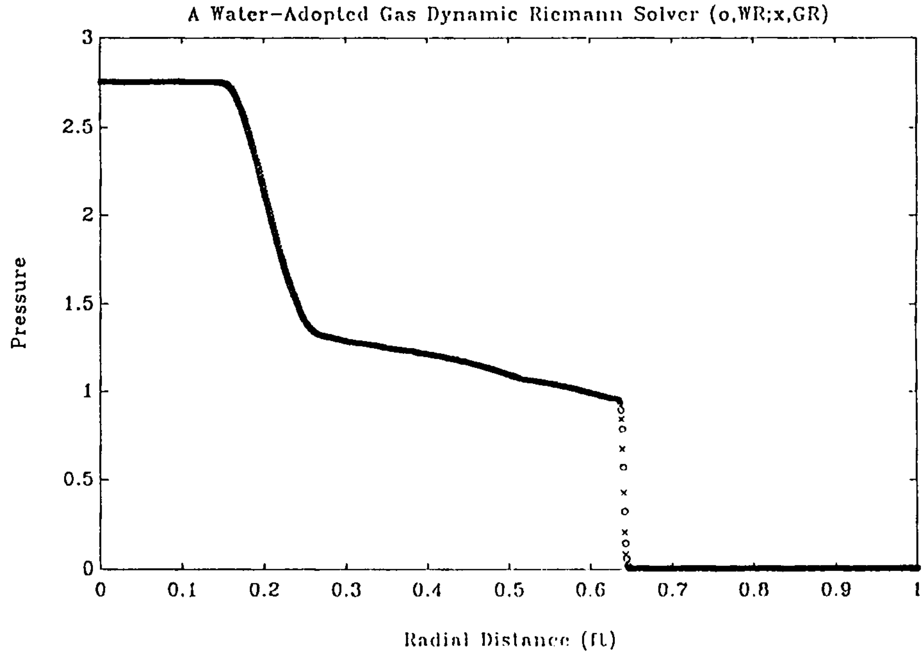

Figure 3. Comparison of results for an exact versus an approximate Riemann solver for water (pressure)

7. CONCLUSIONS

Methods for solving the Riemann problem for water and at a gas–water interface have been given. Numerical results confirm that the modified Tait equation of state allows the classical gas dynamic Riemann solver codes to be readily adapted to problems involving hydrodynamics.

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